AN ROINN OIDEACHAIS AGUS EOLAÍOCHTA

LEAVING CERTIFICATE EXAMINATION, 2001

APPLIED MATHEMATICS – ORDINARY LEVEL

FRIDAY, 22 JUNE – AFTERNOON, 2.00 to 4.30

Six questions to be answered. All questions carry equal marks.

Mathematics Tables may be obtained from the Superintendent.

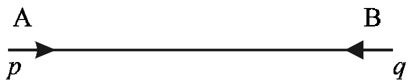
Take the value of g to be 10 m/s^2.

$\vec{i}$ and $\vec{j}$ are unit perpendicular vectors in the horizontal and vertical directions, respectively, or eastwards and northwards, respectively, as appropriate to the question.

Marks may be lost if necessary work is not clearly shown or you do not indicate where a calculator has been used.

---

1. Two points, p and q, lie on a straight stretch of level road.
Car A passes the point p with a speed of 2 m/s travelling towards q and accelerating uniformly at 2 m/s^2.
As car A passes p, car B passes the point q with a speed of 1 m/s travelling towards p and accelerating uniformly at 3 m/s^2 .
The two cars meet after 10 seconds.

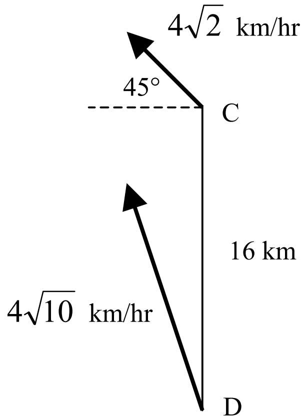


(i) Find the speed of each car when they meet.

(ii) Find the distance each car has travelled during this 10 seconds.

Suppose now that the speed of car A when passing point p is u m/s instead of 2 m/s, while the speed of car B passing point q and the acceleration of each car remain unchanged.
If the time taken for the two cars to meet in this case is 8 seconds, find the value of u.

2. At a certain instant ship D is 16 km due south of ship C.
Ship C is travelling with a speed of $4\sqrt{2}$ km/hr in a north-westerly direction.
Ship D is travelling with a speed of $4\sqrt{10}$ km/hr to intercept C.
Let the velocity of D be $x\vec{i} + y\vec{j}$ km/hr.



(i) Write down the velocity of C in terms of $\vec{i}$ and $\vec{j}$.
(ii) Find the value of x and the value of y.
(iii) How long does it take ship D to intercept ship C ?

3. A straight vertical cliff is 45 m high.
Projectile P is fired horizontally directly out to sea from the top of the cliff with a speed of 20 m/s.

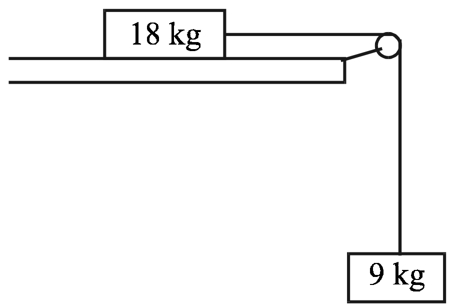
How long does it take projectile P to hit the sea ?
At what distance from the foot of the cliff does projectile P hit the sea ?

Projectile Q is also fired directly out to sea from the top of the cliff with a velocity of $x\vec{i} + y\vec{j}$ m/s, that is, with horizontal velocity component of x m/s and vertical velocity component of y m/s.
Projectile Q takes twice as long to hit the sea as projectile P did.
Projectile Q hits the sea three times as far from the foot of the cliff as projectile P did.

Show that the value of x is 30 and find the value of y.

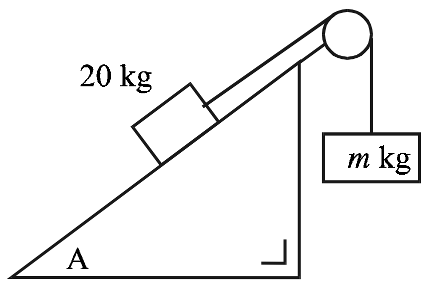
4. **(a)** Two particles, of masses 18 kg and 9 kg respectively, are connected by a light inextensible string passing over a smooth light pulley at the edge of a rough horizontal table. The coefficient of friction between the 18 kg mass and the table is μ.

The 9 kg mass hangs freely under gravity.
The particles are released from rest.
The 9 kg mass moves vertically downwards with an acceleration of $\frac{5}{9}$ m/s^2.



(i) Show on separate diagrams all the forces acting on each particle.
(ii) Find the value of the tension in the string.
(iii) Find the value of μ, giving your answer as a fraction.

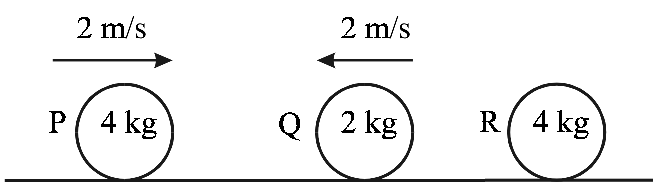
(b) A particle of mass 20 kg is placed on a rough plane inclined at an angle A to the horizontal where tan A = $\frac{3}{4}$. This particle is connected by means of a light inextensible string passing over a smooth light pulley at the top of the plane to a particle of mass m kg, hanging freely under gravity.
The coefficient of friction between the 20 kg mass and the plane is $\frac{1}{4}$.



The system is released from rest. The 20 kg mass moves up the plane.
The value of the tension in the string is 200 Newtons.

(i) Find the common acceleration of the particles.
(ii) Show that $m = 25$.

5. A smooth sphere P, of mass 4 kg, moving with a speed of 2 m/s collides directly with a smooth sphere Q, of mass 2 kg, travelling in the opposite direction with a speed of 2 m/s on a smooth horizontal table.
The coefficient of restitution for the collision is $\frac{1}{3}$.

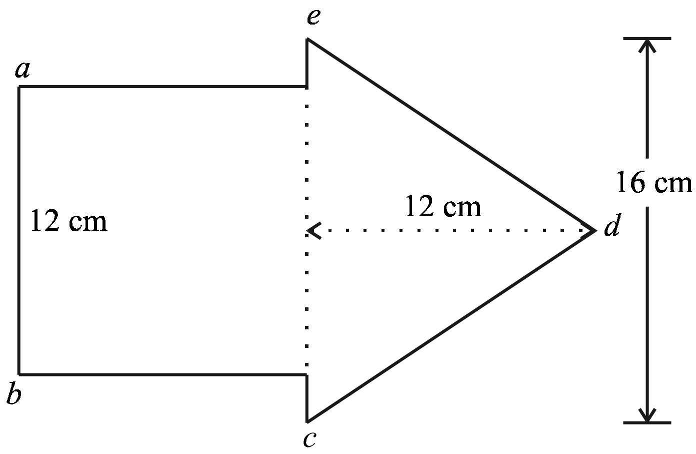


Find the speed of P and the speed of Q after the collision.

As a result of this collision Q goes on to collide directly with a stationary smooth sphere R, of mass 4 kg. The collision between Q and R causes Q to come to rest.

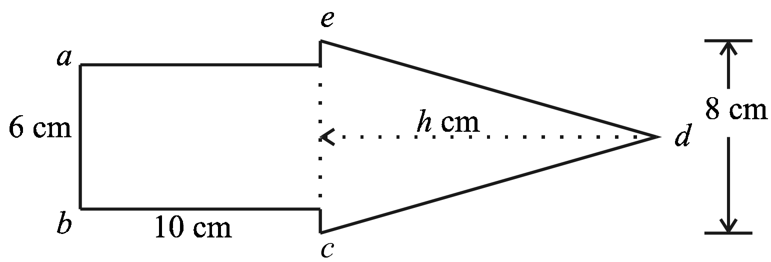
Find the coefficient of restitution for the collision between Q and R.

6. An arrow shape sign, made of uniform plastic, is to be designed to indicate the direction to the office in a school. The diagram shows the initial design, a square with side of length 12 cm and triangle cde with $|ce| = 16$ cm. The distance from d to $[ce]$ is 12 cm.



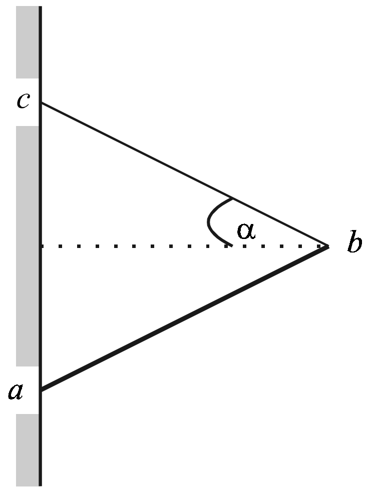
Find the distance of the centre of gravity from $[ab]$.

The designer is unhappy with the shape and redesigns the sign as follows: a rectangle with sides of length 6 cm and 10 cm and triangle cde with $|ce| = 8$ cm. The distance from d to $[ce]$ is h cm. The centre of gravity of this new design lies on $[ce]$.



Find the value of h.

7. One end of a uniform rod $[ab]$, of length $2l$ and mass 20 kg, is freely hinged to a smooth vertical wall at a. The other end b is supported by a light inextensible string tied to a point c on the wall vertically above a. The string $[bc]$ and the rod $[ab]$ have equal length. The string is inclined at an angle α to the horizontal where $\tan\alpha = \dfrac{1}{2}$.

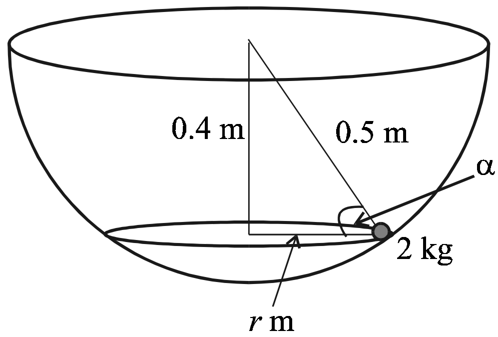


(i) Show on a diagram all the forces acting on the rod $[ab]$.

(ii) Find the horizontal and vertical components of the reaction at a.

(iii) Show that the tension in the string is $50\sqrt{5}$ N.

8. A smooth particle of mass 2 kg describes a horizontal circle of radius r metres with constant angular velocity ω radians per second on the smooth inside surface of a hemispherical bowl of radius 0.5 metres.
The centre of the horizontal circle is 0.4 metres vertically below the centre of the circle formed by the rim of the bowl.
The normal reaction between the particle and the bowl makes an angle α with the horizontal.



(i) Find the value of r.
(ii) Write down the value of $\cos\alpha$ and of $\sin\alpha$.
(iii) Show on a diagram all the forces acting on the particle.
(iv) Find the normal reaction between the particle and the bowl.
(v) Find the value of ω.

9. State the principle of Archimedes.

A solid rectangular block measures 2 m × 2.5 m × 1.2 m.
The block floats at rest in water.
Its shortest edge, of length 1.2 m, is vertical.
Half of its volume is immersed in the water.

The density of the block is ρ kg/m^3.

Find the value of ρ.

The block is now turned in the water so that it floats with its longest edge, of length 2.5 m, vertical.

Find what fraction of the volume of the block is now immersed in the water.

[Density of water = 1000 kg/m^3.]